Minitab®

SPC

LESSON: Quality Methods - Control Charts for Attributes

Quality Methods

Control Charts for Attributes

Advantages and Disadvantages

Chart for Proportion Nonconforming (*p*-chart)

Chart for Number Nonconforming (*np*-chart)

Dr. Diane Evans

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY

Commonly Used Control Charts

- **Variables data** (measurement data, continuous)
  - ◊ $\bar{X}$ and R-charts
  - ◊ $\bar{X}$ and s-charts
  - ◊ Charts for individuals (*I-MR*)

- **Attribute data** (count data, discrete)
  - ◊ **For number or proportion of "defective" units in a lot:**

    ***p*-chart, *np*-chart**

  - ◊ For number of "defects" per unit:

    ***c*-chart, *u*-chart**

Charts for Variables vs. Attributes

- Variables (continuous)
  - ◊ **Quantitative**
  - ◊ Both mean and variation charts
  - ◊ Higher sampling cost in general
  - ◊ Identify mean shifts sooner before large number nonconforming
  - ◊ Used at the lower levels (floor) as a production tool

- **Attributes (discrete)**
  - ◊ **Qualitative**
  - ◊ Must define upfront what a non conformity is for that process
  - ◊ Larger sample sizes may be needed to detect nonconformities
  - ◊ Provides summary level performance
  - ◊ High level (manager) summary tool

Attribute Charts: The Goods & Bads

- **Advantages**
  - ◊ Some things best measured as attributes
  - ◊ If many characteristics to measure (length, height, width) – can reduce cost with attribute measure e.g. Bigger than a bread box?
  - ◊ Attributes occur at all levels in organization, variable charts typically used at lowest level.

- **Disadvantages**
  - ◊ No indication of degree to which specifications are met
  - ◊ Do not get mean and variability info
  - ◊ May not tell process changes as soon as variable chart does
  - ◊ May require larger sample sizes

Considerations for Using Attribute Charts

- **Need to consider:**
  - ◊ Sampling interval,
  - ◊ Sample size,
  - ◊ Sample size guideline, e.g., $np > 10$

- The One, Two, and Three Sigma Zones and Rules pertaining to variable control charts ***may not*** apply to attribute charts – why not?

Types of Attribute Charts

1. Charts for proportion or number of defective units per lot based on the **binomial distribution**. You **MUST** have a **finite sample size *n*** for these charts.

p & np charts

2. Charts for nonconformities, or number of defects, **per unit** (time, space, area) based on the **Poisson distribution** with no sample size given:

c & u chart

	Number of defective units per lot	Number of defects per unit
Constant Sample Size	***np – chart***	***c – chart***
Varying Sample Size	***p – chart***	***u – chart***
Distribution	**Binomial**	**Poisson**

Sample Exercise

In a process manufacturing small fruit cakes (containing cherries, currants and raisins) the following are routinely monitored.

(a) The number of burnt cakes in a box of 12 cakes

(b) The number of raisins in a cake

(c) The weight of a cake

Which distributions (and thus control charts) could be used to model these observations?

p, np **- charts**

p is the proportion of defective units per lot
np is total number of defective units per lot

- For control charts based on the binomial distribution, certain assumptions must be met:
  - ◊ Probability of defective items per lot is assumed to be **constant** from lot to lot.
  - ◊ The lots are “identical” and are assumed to be independent
- Sample size must be large enough. Why?
- Clearly define what constitutes a defective unit in a lot
  - ◊ Example: Scratch on watch face vs. scratch on machine vise

Example 1. The number of nonconforming cables is found for $k = 20$ samples of size $n = 100$ and is shown below. The data are **Lesson19DATA_P_ControlChart**. Construct a control chart for the proportion of nonconforming cables per lot.

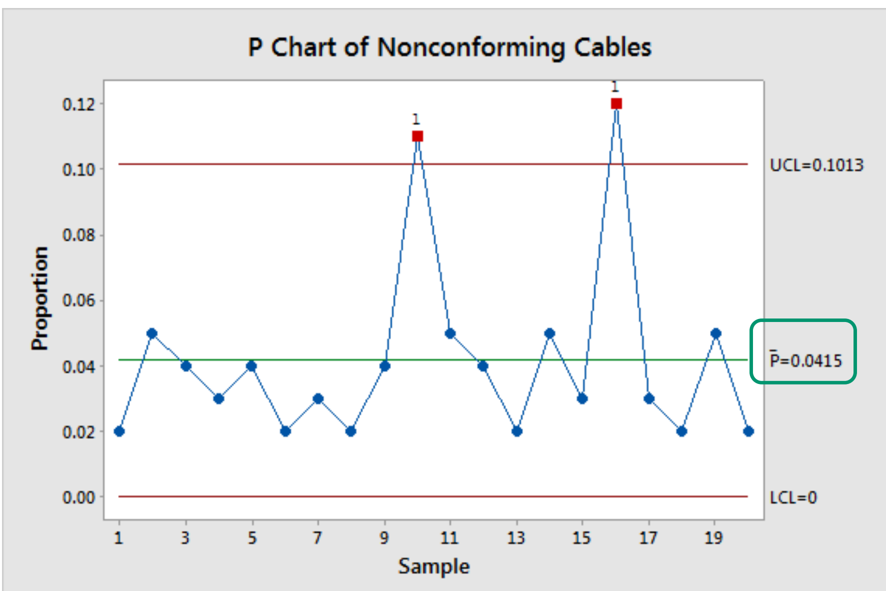
Sample	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Defective units per lot	2	5	4	3	4	2	3	2	4	11	5	4	2	5	3	12	3	2	5	2

The proportions plotted on the *p*-chart are:

$$\hat{p}_1 = 0.02,\ \hat{p}_2 = 0.05,\ \hat{p}_3 = 0.04, \ldots,\ \hat{p}_{20} = 0.02$$

Minitab:

Choose **Stat** > **Control Charts** > **Attribute Charts** > **P**.
In **Variables**, enter *Nonconforming Cables*.
In **Subgroup sizes**, enter *100*.
Click **OK**.

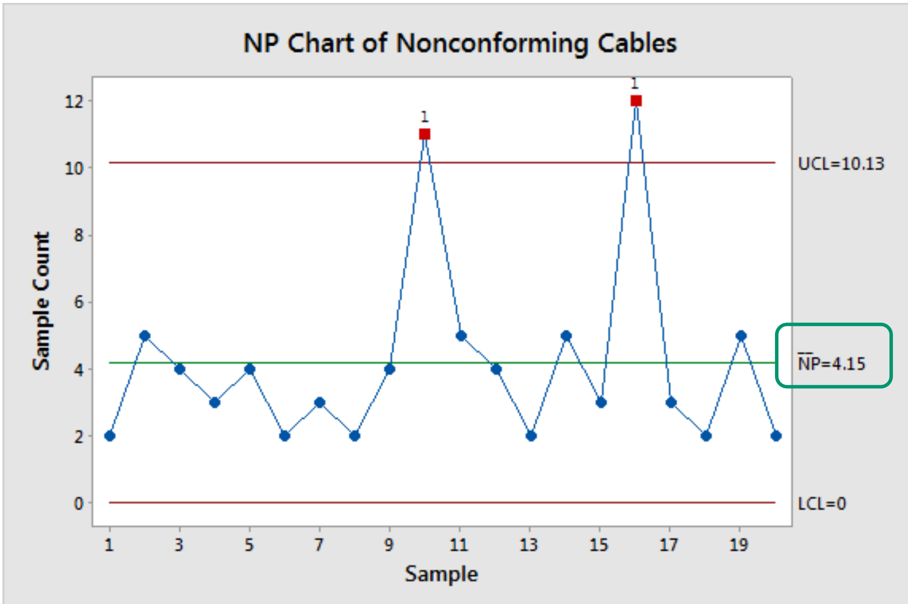


np-chart

- Instead of calculating proportion of nonconforming per lot, we count the number of nonconforming per lot
- Operating personnel sometimes relate to number of nonconforming better than proportion of nonconforming
- Looks identical to p chart when sample size n is constant; only difference is chart tracks the number of defectives per sample or lot

Minitab:

Choose **Stat** > **Control Charts** > **Attribute Charts** > **NP**.
In **Variables**, enter *Nonconforming Cables*.
In **Subgroup sizes**, enter *100*.
Click **OK**.



Control chart for proportion of defective units: p-chart

- Proportion of defective units per lot i of size n is:

$$\hat{p}_i = \frac{x_i}{n_i}$$

x_i = number of defective units in lot i

n_i = lot size of sample i

For k samples:

$$\bar{p} = \frac{\sum_{i=1}^{k} \hat{p}_i}{k}$$

Control Charts Limits for *p*-chart

$$\bar{p} = \frac{\sum_{i=1}^{k} \hat{p}_i}{k}$$

Standard deviation of a binomial distribution X with probability of defectives $\bar{p}$ and sample size n:

$$\sigma = \sqrt{n \cdot \bar{p} \cdot (1 - \bar{p})}$$

Standard deviation of $\bar{p}$ with sample size n:

$$s_{\bar{p}} = \frac{\sqrt{n \cdot \bar{p} \cdot (1 - \bar{p})}}{n} = \sqrt{\frac{\bar{p} \cdot (1 - \bar{p})}{n}}$$

$$UCL_p = \bar{p} + 3 \cdot s_{\bar{p}} \qquad LCL_p = \max(0, \bar{p} + 3 \cdot s_{\bar{p}})$$

15

Control Charts Limits for *np*-chart

$$CL_{np} = n \cdot \bar{p}$$

$$UCL_{np} = n \cdot \bar{p} + 3 \cdot \sqrt{n \cdot \bar{p} \cdot (1 - \bar{p})}$$

$$LCL_{np} = \max(0, n \cdot \bar{p} - 3 \cdot \sqrt{n \cdot \bar{p} \cdot (1 - \bar{p})})$$

***p*-chart when standards p_0 and *n* are provided:**

Target value for proportion defective per lot: p_0

$$s_{p_0} = \sqrt{\frac{p_0 \cdot (1 - p_0)}{n}}$$

$$UCL_{p_0} = p_0 + 3 \cdot s_{p_0} \qquad LCL_{p_0} = \max(0, p_0 + 3 \cdot s_{p_0})$$

16

Example: *p* chart with variable *n*: